A Probabilistic Model of the Radar Signal-to-Clutter and Noise Ratio for Weibull-Distributed Clutter

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Abstract-We consider four effects relevant to the determination of the ratio of radar signal to clutter and noise. These effects are atmospheric turbulence, target fluctuations based on the Swerling models, zero-mean Gaussian background and receiver noise, and Weibull-distributed clutter. Radar return signal levels are affected by target fluctuations and atmospheric turbulence, characterized by target fluctuations according to the Swerling models and a lognormal distribution, respectively. Since these distributions are not independent and identically distributed (IID), they cannot be simply added, and must be treated by combining them in a manner similar to convolution. Also, clutter and noise are not IID, and must be combined in a similar way. The ratio of these two combinations comprises a probabilistic model of the ratio of radar signal to clutter and noise. This ratio is the probability that a given signal level will be achieved in the presence of atmospheric and target scintillations divided by the probability that a given clutter and noise level will be observed. To determine the ratio of the actual signal to clutter and noise, we must multiply these probabilities by the mean powers resulting from these phenomena, as will be shown later. We treat several cases of interest by varying the average radar cross section, the log intensity standard deviation of turbulence, the radar threshold-to-noise and signal-to-noise ratios, and the distribution of Weibull clutter mentioned above.

I. INTRODUCTION

Atmospheric turbulence manifests itself in many ways in electromagnetic propagation. The most common effect is that of intensity fluctuations commonly observed in many readily observable situations, such as the twinkling of stars [1]. It is this manifestation that is considered in this paper. Other effects are phase fluctuations that result in scintillation of the angle of arrival of a beam. Still others result in thermal blooming and related phenomena.

In a similar way, radar targets have been shown to fluctuate significantly as a function of angle. Very small changes in angle can give rise to changes in radar cross section of an order of magnitude or more. The seminal work of Swerling [2] in the study of radar cross sections has resulted in the elegant and simple models of cross section

fluctuations commonly used today. The numerator of the expression for the ratio of the radar signal to clutter and noise (SCNR) comprises the combination of turbulent and target fluctuations.

Noise in a radar receiver comes from background thermal radiation as well as noise generated in the receiver itself. This noise has almost always been treated as a zero-mean Gaussian process. Radar clutter comes from undesired scatter from objects within the radar beam that are not targets, and has been characterized by several distributions, including lognormal, Weibull, Rayleigh, and normal as well as a few others. The parameters of these distributions are chosen based on the type of clutter in the beam, for example urban, wooded, sea, and cropland scenarios, as well as the radar signal grazing angle. The denominator of the expression for the ratio of radar signal to clutter plus noise combines the contributions of noise and clutter.

The procedure for determining the SCNR follows closely that used by McMillan and Barnes [3], who considered the detection of optical pulses in the presence of Gaussian noise and atmospheric scintillation. McMillan [4] used a similar approach to calculate the probability of achieving a given minimum power density on a target in a laser weapon scenario, and McMillan and Kohlberg [5], in unpublished work, considered the probability of achieving a minimum power on a radar receiver for fluctuating targets in atmospheric scintillation. Related work was done by Farina, Russo, and Studer [6], who studied radar detection in lognormal clutter, and by Morgan, Moyer, and Wilson [7], who developed a method for determining the optimal radar threshold in Weibull clutter and Gaussian noise. More recently, McMillan and Kohlberg [8] used a very similar approach to determine the SCNR for lognormal clutter.

II. THEORY: DETERMINATION OF THE SIGNAL LEVEL

We consider the theory used to develop the probabilistic SCNR in two parts: Part 1 develops the signal (numerator) part of this ratio, and Part 2 derives the clutter and noise portion (denominator). For the case treated in this

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14. ABSTRACT
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paper, the size of the electromagnetic beam is assumed to be much larger than the size of the aperture of the radar, a restriction that is generally met for radar wavelengths. We also assume that the time duration of the transmitted signal or coherent processing interval is short compared to the time required for the atmospheric turbulence, noise, target, and clutter statistics to change and that these statistics do not change appreciably over the small angles over which the target fluctuations change. These conditions are generally met for microwave or millimeter-wave radars. Target and atmospheric scintillations must also be uncorrelated. The numerator is the probability $P(R)$ of receiving a given power W_R required to achieve a predetermined signal to noise ratio for a Swerling I target and is [4,5,9]

$$P(R) = \frac{1}{(2\pi)^{\frac{1}{2}}\sigma_W \sigma_{AV}} \int_{\sigma_R}^{\infty} exp\left(-\frac{\sigma}{\sigma_{AV}}\right) d\sigma$$

$$\cdot \int_{W_R}^{\infty} exp\left[-\frac{(lnW - lnW^*)^2}{2\sigma_W^2}\right] d(lnW), \qquad (1)$$

where σ_W is the log intensity standard deviation of turbulence, σ_{AV} is the average radar cross section of the target, σ_R is the radar cross section required to give $P(R)$, and lnW^* is the mean of lnW [10]. In this equation, the first integral characterizes Swerling I target fluctuations and the second relates to the log normally distributed atmospheric scintillations. The Swerling III target will be treated later. The required power W_R is related to the required radar cross section by the equation $W_R = C\sigma_R$, where C is a constant that accounts for all of the parameters of the radar range equation. This expression says simply that the return power is proportional to the radar cross section. Similarly, $W = C\sigma$, and $d\sigma = (1/C)dW$. Making these substitutions and performing the first integration gives simply $exp(-W_R/C\sigma_{AV})$. In the second integral, let

$$Y = \frac{lnW - lnW^*}{\sqrt{2}\sigma_W}, \qquad (2)$$

then $lnW = \sqrt{2}\sigma_W Y + lnW^*$, and $dln(W) = \sqrt{2}\sigma_W dY$. At $W = W_R$, $Y = (lnW_R - lnW^*)/\sqrt{2}\sigma_W$, and at $W = \infty, Y = \infty$. Making these substitutions gives

$$P(R) = exp\left(-\frac{W_R}{C\sigma_{AV}}\right)$$

$$\cdot \frac{1}{\sqrt{\pi}} \int_{(lnW_R - lnW^*)/\sqrt{2}\sigma_W}^{\infty} exp(-Y^2)dY. \qquad (3)$$

The parameter W^* is given by $W^* = \overline{W} \, exp(-\sigma_W^2/2)$ where $\overline{W}$ is the average value of W [10]. Now let $\overline{W} = KW_R$, where K is the fraction of the power W_R required for some level of nominal performance. For example, if W_R is the power required to result in a given signal-to-noise ratio for the radar cross section σ_{AV}, then

$$W^* = KW_R exp\left(-\sigma_W^2/2\right),$$

and $lnW_R - lnW^* = \sigma_W^2/2 - lnK, \qquad (4)$

so that the probability of receiving the required power W_R becomes

$$P(R) =$$

$$exp(-W_R/C\sigma_{AV}) \frac{1}{\sqrt{\pi}} \int_{\left(\frac{\sigma_W^2}{2}-lnK\right)/\sqrt{2}\sigma_W}^{\infty} exp(-Y^2)dY. \qquad (5)$$

The average power received $\overline{W}$ is related to the average radar cross section by $\overline{W} = C\sigma_{AV} = KW_R$, since the radar is designed to return nominal power for a nominal radar cross section. Also note that the integral in this equation is proportional to the complementary error function defined by

$$erfc(x) = 1 - erf(x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} exp(-t^2) \, dt, \qquad (6)$$

so that the final result for the numerator of the SCNR expression for Swerling I targets is

$$P(R) = \frac{exp\left(-\frac{1}{K}\right)}{2} erfc\left[\frac{1}{\sqrt{2}\sigma_W}\left(\frac{\sigma_W^2}{2} - lnK\right)\right]. \qquad (7)$$

In this equation the average radar cross section is implicit in the relation $\sigma_{AV} = \frac{K}{C}W_R$. We can use Equation (7) to calculate the probability of detecting a radar signal from a fluctuating target in the presence of atmospheric turbulence of log standard deviation σ_W as a function of the ratio K of power transmitted to that required to detect this signal in the absence of target and atmospheric fluctuations for an average radar cross section. Such a plot is shown in Figure 1 for $\sigma_W = 0.0, 0.2, 0.5$ and 0.7, values ranging from light turbulence to heavy turbulence.

We now consider Swerling III target fluctuations, characterized by

$$P(\sigma) = \frac{4\sigma}{\sigma_{AV}^2} exp(-2\sigma/\sigma_{AV}). \qquad (8)$$

The probability that the radar cross section will be greater than some value σ_R required to give nominal performance is

$$P(\sigma_R) = \frac{4}{\sigma_{AV}^2} \int_{\sigma_R}^{\infty} \sigma exp(-2\sigma/\sigma_{AV})d\sigma. \qquad (9)$$
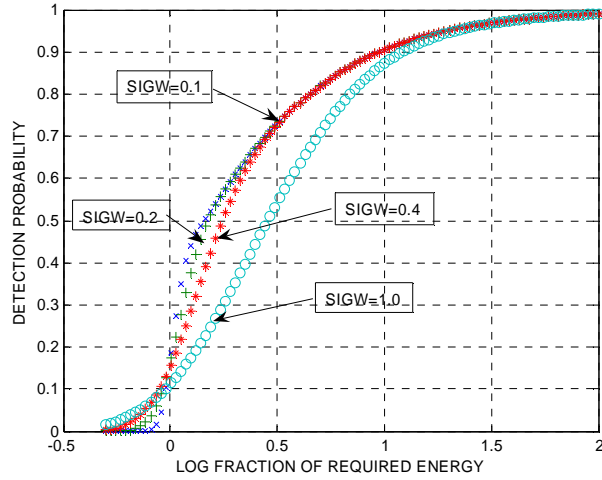
Figure 1. Detection probability for a Swerling I target as a function of nominal energy required for detection in atmospheric turbulence characterized by different values of σ_W (SIGW).

Integration gives $\left(\frac{2\sigma_R}{\sigma_{AV}}+1\right)exp(-2\sigma_R/\sigma_{AV})$. Using $\sigma_R = E_R/C$ and $\sigma_{AV} = KE_R/C$, we get

$$P(\sigma_R) = \left(\frac{2}{K}+1\right)exp(-2/K). \qquad (10)$$

This expression multiplies the erfc expression in (7) above. The final result for the numerator of the SCNR expression for the Swerling III case is then

$$P_{III}(\sigma_R) =$$

$$\left(\frac{2}{K}+1\right)exp(-2/K)erfc\left[\frac{1}{\sqrt{2}\sigma_W}\left(\frac{\sigma_W^2}{2}-lnK\right)\right]. \qquad (11)$$

Figure 2 shows a plot of the detection probability for a Swerling III target under the same conditions as those of Figure 1.

To obtain the actual probabilistic signal power level, we must multiply (7) and (11) by the nominal power received based on σ_{AV} and the nominal required power level W_R. Using the expression for the mean of the lognormal distribution given in (4), the nominal power is proportional to

$$\sigma_{AV}W^* = \sigma_{AV}\overline{W}exp\left(-\frac{\sigma_W^2}{2}\right) =$$
$$\sigma_{AV}KW_Rexp\left(-\frac{\sigma_W^2}{2}\right). \qquad (12)$$

This expression must be multiplied by (7) or (11) to give the average signal power.
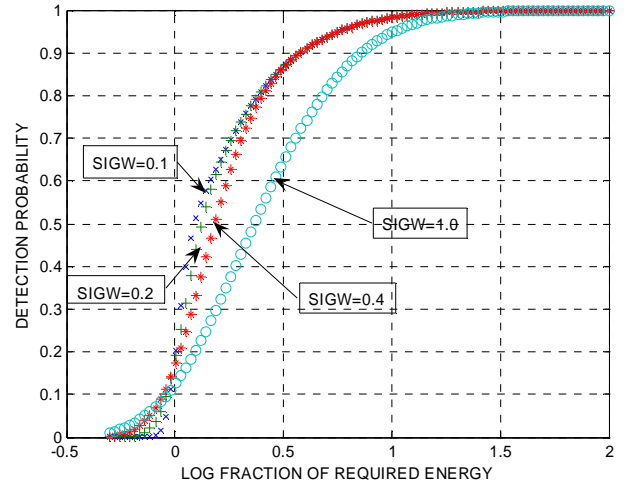
Figure 2. Detection probability for a Swerling III target under the same conditions as those specified for Figure 1.

II. THEORY: DETERMINATION OF THE CLUTTER AND NOISE LEVELS FOR WEIBULL CLUTTER

This portion of the theory combines zero-mean Gaussian receiver and background noise with Weibull clutter. It follows closely the development of the combination of Gaussian noise and lognormally-distributed atmospheric turbulence given in [3] and uses an approach similar to that in the previous section. In this section we develop an expression for the denominator of the signal to clutter and noise ratio for Weibull clutter. The probability that a given threshold T will be exceeded in the presence of Gaussian noise of standard deviation σ_N and Weibull-distributed clutter with shape parameter a and scale parameter b is

$$P(T) =$$
$$\frac{1}{\sqrt{2\pi}\sigma_N}\int_T^\infty \frac{(\tau-U)^2}{2\sigma_N^2}d\tau \cdot \frac{a}{b}\int_T^\infty\left(\frac{x}{b}\right)^{a-1}exp\left[-\left(\frac{x}{b}\right)^a\right]dx, \qquad (13)$$

where U is power. The first integral is simply the complementary error function

$$erfc\left(\frac{T-KU_0}{\sqrt{2}\sigma_N}\right), \qquad (14)$$

where we have used U_0 as the nominal required power and multiplied it by the ratio K as we did earlier. The closed-form solution for the second integral is

$$exp\left[-\left(\frac{T}{b}\right)^a\right]. \qquad (15)$$

In the expression (14), T/σ_N and U_0/σ_N are the threshold- and signal-to-noise ratios chosen for some nominal performance as before. The mean of the Weibull distribution is

$$Mean = b\Gamma(1 + 1/a) = KU_0. \qquad (16)$$

This expression states simply that the mean of the clutter return power is proportional to the transmitted power. It accounts for the fact that more clutter power is returned if more power is transmitted.

Solving for T/b, we get $\frac{T}{b} = \frac{T\Gamma(1+1/a)}{KU_0}$, and since $TNR = T/\sigma_N$, and $SNR = U_0/\sigma_N$, we find

$$\frac{T}{U_0} = \frac{TNR}{SNR}, \text{ and } \frac{T}{b} = \frac{TNR \cdot \Gamma(1+1/a)}{K \cdot SNR}. \qquad (17)$$

The final result for the probability that threshold will be exceeded in Weibull clutter and Gaussian noise is then

$$P(T) = exp\left\{\left[\left(-\frac{TNR}{KSNR}\Gamma\left(1 + \frac{1}{a}\right)\right)\right]^a\right\}$$

$$\cdot erfc\left(\frac{T - KU_0}{\sqrt{2}\sigma_N}\right). \qquad (18)$$

To obtain the probabilistic SCNR for Weibull clutter, we calculate the ratio of the probabilities (7) or (11) to that of (18) and multiply by the ratio R of the signal power to clutter power in the presence of noise and target fluctuations, which is the ratio of the average signal power (12) to the average clutter and noise power $\sigma_{CS}KU_0$ to get:

$$R = \frac{\sigma_{AV}KW_R exp\left(-\frac{\sigma_W^2}{2}\right)}{\sigma_{CS}KU_0} = \frac{\sigma_{AV}}{\sigma_{CS}}exp(-\sigma_W^2/2), \qquad (19)$$

Where we have made use of the fact that $W_R = U_0$.

III. RESULTS AND DISCUSSION

We have plotted the ratio of Equations (7) and (11) to (18) for Swerling I and Swerling II targets in Figures 3 and 4. For these plots, the log standard deviation of the atmospheric turbulence distribution was chosen to be 0.4, $T/\sigma_N = 2$ and $U_0/\sigma_N = 3$, and the Weibull spread parameter a was varied from 1 to 4. Figure 3 shows that there is no further increase in SCNR derived by transmitting more power beyond a value of approximately 10 times the nominal required to give nominal detection probability and false alarm rate. Figure 4 shows the surprising result that the SCNR peaks at roughly three times nominal power for a Weibull shape parameter a of 1.0 and for a Swerling III target and decreases afterward. A similar result was obtained in [8] for lognormal clutter. We must keep in mind that these calculations are based on complex targets that fluctuate according to the Swerling phenomena in

atmospheric turbulence, and would not hold for steady targets such as spheres or corner cubes. In these cases, the signal would increase with power, resulting in an increase in SCNR. For both Swerling I and Swerling III targets, all curves approach the quantity R in (19) as power is increased.
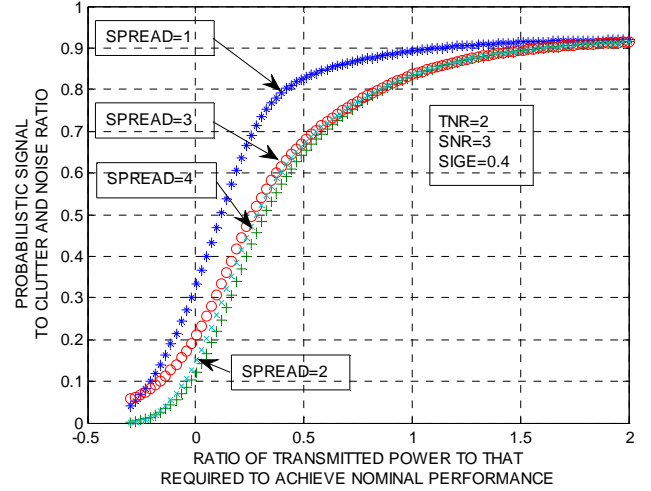
Figure 3. Probabilistic signal to clutter and noise ratio for nominal threshold to noise ratio (TNR) = 2.0, nominal signal to noise ratio (SNR) =3, $\sigma_{AV}/\sigma_C = 1.0$, and log standard deviation of atmospheric turbulence (SIGW) = 0.4 for various values of the Weibull clutter spread parameter a for a Swerling I target.

It is important to point out that these calculations do not consider the considerable gains that can be made by signal processing. In this paper we consider only the signal, clutter, and noise levels.
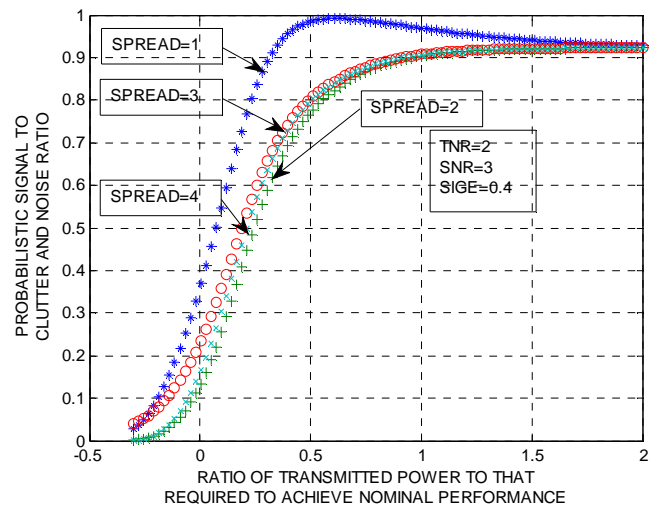
Figure 4. Probabilistic SCNR for a Swerling III target under the same conditions as those of Figure 3.

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